Spin down (?) of protostars through gravitational torques

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12 August 2010
Plan

- Problem
- Numerical modelling
  - Simulation 1: hindered spin down
  - Simulation 2: spin down
- Discussion and implications
- Problems
Angular momentum problem in star formation (Bodenheimer, 1995). Specific angular momenta (cgs) of:

- Molecular cloud core, $j \sim 10^{21}$
- T Tauri, $j \sim 10^{17}$
- Sun, $j \sim 10^{15}$

Need to transport angular momentum out of central region, otherwise star spins too fast. How?
Problem

$t/\text{kyr} = 1.08 \times 10^2, \ | = 0.050L$
Governing equations

Inviscid, non-magnetised and self-gravitating fluid with customised equation of state:

\[
\begin{align*}
\frac{D\rho}{Dt} &= -\rho \nabla \cdot \mathbf{v} \\
\frac{D\mathbf{v}}{Dt} &= -\frac{1}{\rho} \nabla P - \nabla \Phi \\
\rho \frac{De}{Dt} &= -\rho \mathbf{v} \cdot \nabla \Phi - \nabla \cdot (P \mathbf{v}) \\
\nabla^2 \Phi &= 4\pi G \rho, \\
P &= c^2 \rho^{\gamma_1} \left[ 1 + \left( \frac{\rho}{\rho_*} \right)^{\gamma_2 - \gamma_1} \right].
\end{align*}
\]

\(c \approx 266\text{ms}^{-1}\) is isothermal sound speed of \(\mu = 2.33\) gas at 20K, \(\gamma_1 = 1\) and \(\gamma_2 = 5/3\). Here, \(e\) is the sum of internal and kinetic energy densities.
Angular momentum transport

Conservation equation

\[
\frac{\partial}{\partial t} \left( \rho R v_\phi \right) + \nabla \cdot \left( \rho R v_\phi v \right) = -\rho \frac{\partial \Phi}{\partial \phi} - \frac{\partial P}{\partial \phi},
\]

integrate over volume \( V \), use \( \rho = \nabla^2 \Phi / 4\pi G \) on RHS to get:

\[
\frac{\partial J}{\partial t} + \oint F \cdot dS = 0
\]

with

\[
F = F_A + F_G
\]

\[
F_G = \frac{1}{4\pi G} \frac{\partial \Phi}{\partial \phi} \nabla \Phi.
\]

Also have the Reynolds stress, \( \rho R \delta v_\phi \delta v \).
Collapse into star-disc system

Kratter et al. (2010) description of collapse of spherical, rotating cloud into a disc (mass $M_d$) with central object (mass $M_*$). Call $M_* + M_d = M_{\text{sys}}$.

- Infall parameter

$$\xi = \frac{G \dot{M}}{c^3}$$

- Rotation parameter

$$\Gamma = \frac{\dot{M}}{M_{\text{sys}} \Omega_k}$$

$\Omega_k$ is Keplerian frequency, due to $M_{\text{sys}}$, of material joining the system from the cloud, assumed to occur at cylindrical radius $R_k$. Can show disc aspect-ratio

$$h = \left( \frac{\Gamma}{\xi} \right)^{1/3},$$

and $R_k = h^2 \xi c t$. 
Initial conditions & numerical method

- Start with spherical cloud of radius $r_c$ of density profile

\[ \rho(r) = \frac{Ac^2}{4\pi Gr^2} \]

Shu (1977) → self-similar collapse.

- Designate a central region $r \leq r_* \equiv qr_c$ to be the ‘star’. Set $\rho_* = \rho(r_*)$.

- Set $v_r = v_\theta = 0$ and azimuthal velocity

\[ v_\phi = 2Ach \times \begin{cases} 
\frac{R}{r_*} & R \leq r_* \\
1 & R > r_*
\end{cases} \]

- Need $2h\sqrt{A} < 1$ for below break-up speed.
Initial conditions & numerical method

- Solve in Cartesian box of length $L = 4r_c$.
- ORION: Godunov-type code with adaptive mesh refinement.
- Base grid $128^3$, 6 refinement levels (effective highest resolution $8192^3$).
Case 1: $\xi = 5.58$, $h = 0.1$, $q = 0.005$

Density slices:

Star spin
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Density slices:

Mode amplitudes

$\log_{10}|a_m/a_0|$ vs. $(R-R_*)/L$ for $m=1$, $m=2$, and $m=3$. 

$\text{t/kyr}=3.93\times10^2$
Case 1: $\xi = 5.58$, $h = 0.1$, $q = 0.005$

Star spin

Modes evolution
Influence of $m = 1$ modes

- Theoretical studies: Adams et al. (1989); Heemskerk et al. (1992).
- $m = 1$ displaces star from COM (of box)
- Require sufficient disc mass.
- Exchange of *orbital* angular momentum.
Case 1: stellar motion & angular momenta

Estimates:

- $\Omega_{\text{spin}} \sim 1.5 \times 10^{-10}$
- $\Omega_{\text{orb}} \sim 2 \times 10^{-13}$
- $\Omega_{\text{disc}}(R_k) \sim 6 \times 10^{-12}$
- $\Omega_{\text{patt}} \sim (3-5) \times 10^{-12}$
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- Spin down around $t = 500$ kyr with visible $m = 2$.
- Spin down near the end but no visible $m = 2$, although FT $\rightarrow m = 2$ still the main non-axisymmetry near star.
- FT $\rightarrow m = 1$ becomes important in outer region but limited orbital motion.
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Star torque & gravity flux

\[ \nabla^2 \Phi_* = 4\pi G \rho_{\text{star}} \rightarrow \text{get star torque per unit area:} \]

consistent with disc-on-star torques, but...
Star torque & gravity flux

\[ \partial_t J + \oint F \cdot dS = 0 \rightarrow \text{look at radial gravity flux near star} \]

Solid and dashed line: characteristic star size.

- \( \alpha < O(10^{-2}) \) also reported in Kratter et al. (2010) but is SMALL compared to numerical \( \alpha \)!
- Numerical spin down? But why ineffective in Case 1?
Case 3: binary spin down
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Gravity flux into binary

- Gravity $\alpha \sim 2$ from binary $\rightarrow$ torque down
- Numerical $\alpha < 0.5$ in this region
- $\Omega_{\text{spin}} > \Omega_{\rho}$ on circle
Problems

- Spin down using gravity flux? Maybe, but cannot have influences from $m = 1$.
- Need better experiment designs to overcome numerical spin down.
- Binary results are less unconvincing.

Thank you

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References

Bodenheimer P., 1995, ARAA, 33, 199