Strange aspects of locally isothermal astrophysical disks and the stability of magnetized massive disks

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Interests

- Astrophysical fluid dynamics
- Disk-planet interactions
- Self-gravitating disks
- Disk instabilities, large-scale structures
- Magneto-hydrodynamics (new)
- Numerical simulations
- Linear/analytical hydrodynamics/methods
Large-scale structures in circumstellar disks

Example: transition disk around HD 142527

(Fukagawa et al., 2013)

Note the scale: \( O(10^2) \) AU
Large-scale structures in circumstellar disks

Example: transition disk around HD 142527

(Spiral scales: S2 from $\sim 500$AU to $\sim 600$AU

(Christiaens et al., 2014)
Modeling hydrodynamics at large distances

- Can we make simplifications?
- Example: irradiated disks

Radiation hydrodynamic simulations from Stamatellos & Whitworth (2008)
- Temperature does not change much as it is essentially set externally
The locally isothermal disk

- Take this to the *idealized limit* of prescribing the temperature distribution:
  \[ T = T(r). \]

- Tremendous simplification: no energy equation to consider
  \[ \rightarrow \] cheaper numerical simulations

- Example: long term disk-planet simulations
The locally isothermal disk

- Take this to the *idealized limit* of prescribing the temperature distribution:
  $$T = T(r).$$

- Tremendous simplification: no energy equation to consider
  → cheaper numerical simulations

- What are the fundamental consequences?
Angular momentum conservation

Essential to all rotating disk problems:

\[ \frac{\partial J}{\partial t} + \nabla \cdot F = T_{\text{ext}}. \]

- \( J \): angular momentum density
- \( F \): angular momentum flux
- \( T_{\text{ext}} \): external torques
Linear stability 101

- Split the system into equilibrium and deviations
  \[ \Sigma \rightarrow \Sigma_{\text{ref}}(r) + \delta \Sigma(r, t) \]
- Linearized equations \( \rightarrow \) time evolution of deviations or perturbations
Linear stability 101

- Split the system into equilibrium and deviations

\[ \Sigma \rightarrow \Sigma_{\text{ref}}(r) + \delta \Sigma(r, t) \]

- Linearized equations → time evolution of deviations or perturbations

Linearized equations → angular momentum conservation for the perturbations

\[ \frac{\partial J_{\text{pert}}}{\partial t} + \nabla \cdot \mathbf{F}_{\text{pert}} = T_{\text{ext,pert}}. \]

- ‘pert’ quantities associated with perturbations

- Definition not obvious
External torque in linear theory

\[
T_{\text{ext, pert}} = \begin{cases} 
0 & \text{barotropic or adiabatic flow} \\
-\frac{m}{2} \text{Im} \left( \delta \Sigma \xi^* \frac{dc_s^2}{dr} \right) & \text{locally isothermal in 2D} \\
\frac{m}{2} \text{Im} \left[ \rho (\nabla \cdot \xi) \xi^* \cdot \nabla c_s^2 \right] & \text{locally isothermal in 3D}
\end{cases}
\]

- Barotropic: \( p(\rho) \), adiabatic: \( \Delta S = 0 \)
- Locally isothermal: sound-speed \( c_s(r) \) fixed
- \( \xi \): Lagrangian displacement, \( m \): azimuthal wavenumber

\[T_{\text{ext, pert}} \neq 0\]

angular momentum exchange between perturbations and the background disk
Can $T_{\text{ext}, \text{pert}}$ make perturbations grow?

Ignoring angular momentum fluxes,

$$\frac{\partial J_{\text{pert}}}{\partial t} \sim T_{\text{ext}, \text{pert}}.$$

- May have an unstable situation if $T_{\text{ext}, \text{pert}}$ is the same sign as $J_{\text{pert}}$
- Possible for low-frequency disturbances in a disk with temperature decreasing outwards (both torque and angular momemtum are negative)
- Low-frequency: $\delta \Sigma \sim e^{i\omega t}$ and $|\omega| \ll m\Omega$. 

*low-frequency disturbances in a disk with temperature decreasing outwards*
Numerical demonstration

2D, self-gravitating disk with radial structure
Numerical demonstration

2D, self-gravitating disk with radial structure

FARGO simulations:
Angular momentum exchange between the background disk and the spiral:
Extracting angular momentum from the spiral:

\[ T_{\text{ext,pert}} \text{ in action} \]

\[ m=1 \]

\[ \frac{10^7 \text{(Torque)}}{J_{\text{ref}} \Omega_k(R_0)} \]

\[ \text{d}J_{\text{lin}}/\text{d}t \]

\[ \text{Torque due to } \frac{dc_s^2}{\text{d}R} \]
Dependence on the imposed temperature gradient

Fixed sound-speed profile $c_s^2 \propto r^{-q}$.
Three-dimensional simulations

Repeat experiment in 3D

- ZEUS: finite difference, discretized Poisson
- PLUTO: Godunov, Poisson through spherical harmonics
- ZEUS results off-set because of numerical issues at boundary

No growth without imposed temperature gradient
Locally isothermal disks are weird: forcing a temperature gradient permit disturbances to exchange angular momentum with the disk

- Application to protoplanetary disks uncertain
- Be star disks?
- Need to develop a more rigorous theory
Three-dimensional locally isothermal disks are baroclinic

If

\[ T = T(R) \propto c_s^2 \]

Then

\[ R \frac{\partial \Omega^2}{\partial z} = - \frac{\partial \ln \rho}{\partial z} \frac{dc_s^2}{dR} \neq 0 \]

- This *vertical shear* may render the disk unstable
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Axisymmetric simulations by Nelson et al. (2013)
Vertical shear instability requires fast thermal relaxation
Otherwise buoyancy prevents vertical motion

Simulations from Nelson et al. (2013) with
\[
\frac{\partial T}{\partial t} = - \frac{T - T_{\text{init}}}{T_{\text{Relax}} P_{\text{orb}}}.
\]
What is ‘fast’ for VSI?

(Lin & Youdin, in prep.)

- $T_{\text{Relax}} = \beta \Omega^{-1}$
- Applied to Minimum Mass Solar Nebulae
Magnetized massive disks

Example 1: protostellar disk formation

Non-ideal MHD plus self-gravity simulation from Inutsuka et al. (2010)
Magnetized massive disks

Example 2: ‘dead zones’ and layered accretion in protoplanetary disks

(Terquem, 2008)

- Can you have a self-gravitating midplane plus MHD turbulent upper/lower layers?
Previous work

- Self-gravitating
  - gravitational instability (GI)
- Magnetized
  - magneto-rotational instability (MRI)

- Fromang et al. (2004)
  - latest dedicated simulations

Also

- Lizano et al. (2010)
  - flat disk model (no MRI): effect of field on GI
MRI plus GI from scratch

Linear model:
- Axisymmetric shearing box
- Self-gravitating
- Magnetized, initially uniform (both $B_z$ and $B_y$ allowed)
- Isothermal or polytropic
- Ohmic resistivity, can be non-uniform

Questions for adding SG to a magnetized disk:
- Are MRI growth rates affected?
- Is ‘layered’ structure possible (GI at the midplane, MRI at top and bottom)?
- Can SG enhance density perturbations from MRI?

(Lin, 2014)
Upper limit on the field strength for MRI in a massive disk

\[ Q = \frac{\Omega^2}{4\pi G \rho_0} \]  
small \( Q \rightarrow \) strong self-gravity

\[ \beta = \frac{c_s^2}{v_{A0}^2} \]  
small \( \beta \rightarrow \) strong field

\[ \Lambda_0 = \frac{v_{A0}^2}{\eta_0 \Omega} \]  
small \( \Lambda_0 \rightarrow \) strong resistivity
Upper limit on the field strength for MRI in a massive disk

Ideal MHD, polytropic disk ($P \propto \rho^2$), vertical field, need

$$\frac{B_z}{c_s \Omega} \sqrt{\frac{\pi G}{\mu_0}} \ll \sqrt{\frac{15}{16}}$$

to get MRI in strongly self-gravitating disks.
Is there a layered MRI-GI mode?

\[ Q = 0.20, \beta = 100.0, \log \Lambda_0 = -1.0, k_x H = 1.34, \gamma = 0.25 \Omega \]

- magnetic
- gravity
- kinetic
- thermal
Enhancing MRI density perturbations

- No self-gravity
- Small thermal/density perturbation
- With self-gravity
- Large thermal/density perturbation
How SG affects MRI depends on symmetry

gravity-dominated, magnetic-dominated

- MRI can be symmetric or anti-symmetric across $z = 0$
- GI can only be symmetric
Summary and future directions

- MRI-GI interaction requires them to have similar scales
  → need weak MRI so its vertical lengthscale $\sim H$ (cf. Fromang et al., 2004)
- Next step: non-axisymmetric perturbations

Eventual goal:
- Full MHD simulations of self-gravitating disks
- Questions:
  angular momentum transport, effect of MRI turbulence on disk fragmentation
Summary and future directions

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For fun: ‘avoided crossing’ between MRI and GI?

![Graph showing the relationship between $k_sH$ and $Q/Q_0$](image)
The vortex instability in non-isothermal disks

(2014 CITA summer student program)

(Les & Lin, 2015, submitted)

- $\beta \ll 1$: fast cooling (isothermal), $\beta \gg 1$: slow cooling (adiabatic)
- There is an optimal cooling rate to maximize vortex lifetime
References


