Dust-free modeling of dusty protoplanetary disks

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Dusty-gas dynamics in protoplanetary disks

Modeling dust-gas interaction is complicated

(Testi et al., 2014)
A simple framework for dusty-gas dynamics

![Diagram showing the equivalence between isothermal dusty gas disk and textbook hydrodynamics for small particles](image-url)
A simple framework for dusty-gas dynamics

isothermal dusty gas disk equivalent for small particles textbook hydrodynamics

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \\
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla P - \nabla \Phi, \\
\frac{\partial P}{\partial t} + \mathbf{v} \cdot \nabla P = -\Gamma P \nabla \cdot \mathbf{v} + \frac{\Gamma P}{\rho_{\text{gas}}} \nabla \cdot (f_{\text{dust}} t_{\text{stop}} \nabla P).
\]

Looks like gas dynamics


- Dust-gas friction

+ ve particle flux - ve heat flux
A simple framework for dusty-gas dynamics

isothermal dusty gas disk equivalent for small particles
textbook hydrodynamics

\[
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\end{align*}
\]

Looks like gas dynamics


- Recent extensions:
  - Dust diffusion
    \[ \frac{\partial \rho_{\text{dust}}}{\partial t} = \nabla \cdot \left[ DP \nabla \left( \frac{\rho}{P} \right) \right] \]
  - Generalized drag laws
    \[ t_{\text{stop}} \propto \left| \mathbf{v}_{\text{dust}} - \mathbf{v}_{\text{gas}} \right|^{-\beta} \]

(K. Chen & Lin, in prep.; ASIAA Summer Student Program 2017)
A simple framework for dusty-gas dynamics

\[ \begin{align*}
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\end{align*} \]


Purpose

- Understand dusty-gas phenomena
- Discover new dusty analogs of hydrodynamic results
- Fast numerical simulations of protoplanetary disks
Dust-trapping by pressure bumps

- Drag forces cause dust to accumulate at pressure bumps
- Built into simplified dust-gas framework
Dusty gas dynamics

Dust traps at planet gaps

- Enable dusty-gas in PLUTO by hacking its thermal conduction module
- No dust equations explicitly solved

Pressure bump at gap edges $\rightarrow$ radial dust trap
Dust traps at planet gaps

- Enable dusty-gas in \textsc{Pluto} by hacking its thermal conduction module
- No dust equations explicitly solved

- Gap edge unstable to vortex formation $\rightarrow$ azimuthal dust trap
Planetesimal formation via the streaming instability

- **Streaming instability** → dust clumping mediated by mutual dust-gas drag
Streaming instability with one-fluid

\[ \text{St}=0.1, \; \rho_d=3\rho_g, \; K_{x,z}=30 \]

More importantly...

- **Thermodynamic interpretation:**
  unstable because of pressure-density lag (cf. Carnot cycles)
Streaming instability with one-fluid

Numerical simulations

cm-sized particles at 10AU

$\rho_{\text{dust}}/\rho_{\text{gas}}$ shown

mm-sized particles at 10AU

No dust module used
Streaming instability with one-fluid

Global cylindrical geometry

Sl needs $\rho_{\text{dust}} > \rho_{\text{gas}}$. Do particles settle to the midplane to begin with?
Dust settling against hydrodynamic turbulence

Laminar disk

VSI-turbulent disk

100 $\times$ $\rho_{\text{dust}} / \rho_{\text{gas}}$ shown; $t_{\text{stop}} \Omega = 0.001 (\ll H/R)$

Protoplanets born into dust-rich environments?
Dust-rich disk-planet interactions

with J.-W. Chen (NTU)

What happens to planet migration in dusty gas disks?
Dust-rich disk-planet interactions

(Chen & Lin, in prep.)

- Oscillitory torques with sufficient $t_{\text{stop}}$ and/or $\Sigma_{\text{dust}}/\Sigma_{\text{gas}} (=0.5 \text{ here})$
Baroclinic dusty torques

(Chen & Lin, in prep.)

- Perfectly coupled particles $\Rightarrow$ potential vorticity mixing in co-orbital region
- Finite particle size $\Rightarrow$ PV generation
Baroclinic dusty torques

Pressure-density misalignment translates to gas/dust density misalignment in isothermal disks:

\[
\frac{D}{Dt} \left( \frac{\omega_z}{\Sigma} \right) = \frac{1}{\Sigma^3} \nabla \Sigma \times \nabla P \\
= \frac{c_s^2}{\Sigma^3} \nabla \Sigma_{\text{dust}} \times \nabla \Sigma_{\text{gas}}
\]

(Laibe & Price 2014)

\[
\frac{\partial}{\partial t} \left( \nabla \Sigma \times \nabla P \right)_{\text{drag}} = c_s^2 \nabla \Sigma \times \nabla \left[ \nabla \cdot \left( f_{\text{dust}} t_{\text{stop}} \nabla P \right) \right]
\]

(Chen & Lin, in prep.)
Summary

- Isothermal dusty gas disk equivalent for small particles
  - Hydrodynamics + cooling


- Dust-drag phenomenon have thermodynamic interpretations
- Dusty analogs of known hydrodynamic effects?
  - Rossby wave/vertical shear/baroclinic instabilities, convective overstability, zombie vortices...etc.
  - Thermodynamic torques in gas disks
- Planet migration and eccentricity evolution in dust-rich disks?
Opportunities at ASIAA

Faculty Positions in Observation, Theory, or Instrumentation

Closing Date: January 15, 2018

Please submit the [online application form]

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