How to fragment protostellar disks with your bare hands

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Wide orbit planets

(Marois et al., 2010)
Disk instability theory

- Young, massive protoplanetary disks fragment under its own gravity

(Rice et al., 2005)
When do protostellar disks fragment on a computer?

\[
Q \equiv \frac{c_s \Omega}{\pi G \Sigma} \lesssim 2 \quad \text{or} \quad \frac{M_{\text{disk}}}{M_*} \gtrsim 0.1
\]

Massive disk

Fast cooling

\[
t_{\text{cool}} \Omega \lesssim "3"
\]

The cooling criterion is empirical.

(Paardekooper, 2012)
When does a real protostellar disk fragment?

If you don’t want to run expensive simulations, then:

1. Work out disk structure: surface density $\Sigma$, temperature $T$
   (This might include physics such as: turbulence, stellar irradiation, radiative cooling...etc.)

2. Is Toomre $Q \sim 1$?

3. Is $t_{\text{cool}} \Omega \sim 1$?
When does a real protostellar disk fragment?

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1. Work out disk structure: surface density $\Sigma$, temperature $T$  
   (This might include physics such as: turbulence, stellar irradiation, radiative cooling...etc.)
2. Is Toomre $Q \sim 1$?
3. Is $t_{\text{cool}} \Omega \sim 1$?

Possible issues:

- Need to choose critical values (mass, cooling rate)
- Complex physics were not included in the numerical experiments that established those values
Experimental uncertainties

What is $t_{\text{cool,crit}}$?

- Meru & Bate (2011): $t_{\text{cool,crit}}$ increases with numerical resolution!
- Numerical details matter! (Lodato & Clarke, 2011; Meru & Bate, 2012; Rice et al., 2014; Young & Clarke, 2015)
Experimental uncertainties

What is $t_{\text{cool,crit}}$?
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- Numerical details matter! (Lodato & Clarke, 2011; Meru & Bate, 2012; Rice et al., 2014; Young & Clarke, 2015)

Does the concept of a $t_{\text{cool,crit}}$ even make sense?
- Paardekooper (2012): just wait for it! Stochastic in nature
- Hopkins & Christiansen (2013): fragmentation is statistical
Conceptual approach

(Rice et al., 2011)
Conceptual approach

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- Write down a classic, viscous disk model to describe quasi-steady, gravito-turbulent state, include cooling physics
- Analyze stability properties
Conceptual approach

(Rice et al., 2011)

- Write down a classic, viscous disk model to describe quasi-steady, gravito-turbulent state, include cooling physics
- Analyze stability properties
- Look for parameter regimes where model breaks down → fragmentation
Fragmentation by cooling

- Cooling removes pressure support against gravity, but how fast should it be?
- Look at dispersion relation for growth rate $s(k)$ and wavenumber $k$

**Classic result** (without cooling)

$$s^2 = 2\pi G\Sigma |k| - \Omega^2 - \gamma c_s^2 k^2$$

Growth = + gravity − rotation − pressure
Fragmentation by cooling

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New result **with cooling**

$$s^2 = 2\pi G\Sigma |k| - \Omega^2 - \left( \frac{T_{irr}/T_0 + \gamma t_{cool} s}{1 + t_{cool} s} \right) c_s^2 k^2$$

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Growth = + gravity − rotation − pressure

- Dispersion relation changes from $s^2 \rightarrow s^3$
- Cooling changes the fundamental nature of the problem
- Condition for stability depends on irradiation temperature $T_{\text{irr}}$
- Can be formally unstable for any $t_{\text{cool}} < \infty$

Just another way to compare compressional heating v.s. thermal losses
A nice result

Special case:

\[ s_{\text{max}} \propto t_{\text{cool}}^{-1/3} \]

Define \( t_{\text{cool,*}} \) as

the cooling timescale to remove pressure over a lengthscale \( \sim \) disk thickness
A nice result

Special case:

\[ s_{\text{max}} \propto t_{\text{cool}}^{-1/3} \]

Define \( t_{\text{cool},*} \) as

\[ t_{\text{cool},*} = (\sqrt{\gamma} - 1)^{-3/2} \Omega^{-1} \]

\begin{center}
\begin{tabular}{c|c|c|c}
\( \gamma \) & \( t_{\text{cool},*} \Omega \) & Simulation, \( t_{\text{cool},\text{frag}} \Omega \) & Reference \\
\hline
7/5 & 12.75 & 12—13 & Rice et al. (2005) \\
1.6 & 7.33 & 8 & Rice et al. (2011) \\
5/3 & 6.37 & 6—7 & Rice et al. (2005) \\
2 & 3.75 & 3 & Gammie (2001) \\
\end{tabular}
\end{center}
Fragmentation by viscosity

Classic instability condition

\[ 2\pi G \Sigma |k| - \Omega^2 - \gamma c_s^2 k^2 > 0 \]

+ gravity − rotation − pressure > 0
Fragmentation by viscosity

Viscous instability condition

\[ 2\pi G \Sigma |k| - c_s^2 k^2 > 0 \]

+ gravity  
- pressure > 0

(Lynden-Bell & Pringle, 1974; Willerding, 1992; Gammie, 1996)

- Viscosity or frictional forces remove rotational stabilization (e.g. inwards migration of particles due to gas-dust drag)
- Model: use \( \alpha \)-viscosity to mimic turbulence
- Simulations: Rice et al. (2005) report a \( \alpha_{\text{max}} \sim 0.1 \) before fragmentation, also supported by Clarke et al. (2007)
Application to protoplanetary disks

- Input physical disk model into stability calculation — get growth timescales

Beyond $\sim 60\text{AU}$:

- Cooling criterion $\Rightarrow$ both disk fragments
- Viscosity criterion $\Rightarrow$ high $\dot{M}$ disk fragments
Application to protoplanetary disks

- Input physical disk model into stability calculation — get growth timescales

Beyond $\sim 60$ AU:

- Cooling criterion $\Rightarrow$ both disk fragments
- Viscosity criterion $\Rightarrow$ high $\dot{M}$ disk fragments, growth times $\sim$ one orbit
Summary

- Analyze the stability properties of a model for non-fragmenting disks (2D/3D shearing box, viscosity, energy equation, optically-thin cooling or radiative diffusion, irradiation)
- Dynamical instability \( \rightarrow \) fragmentation
- Application to physical disk models, determine where and why fragmentation occurs
- Minimal input from numerical simulations

Lin & Kratter, submitted

where and why
References

Willerding E., 1992, Earth Moon and Planets, 56, 173