Dusty protoplanetary disks: a novel approach

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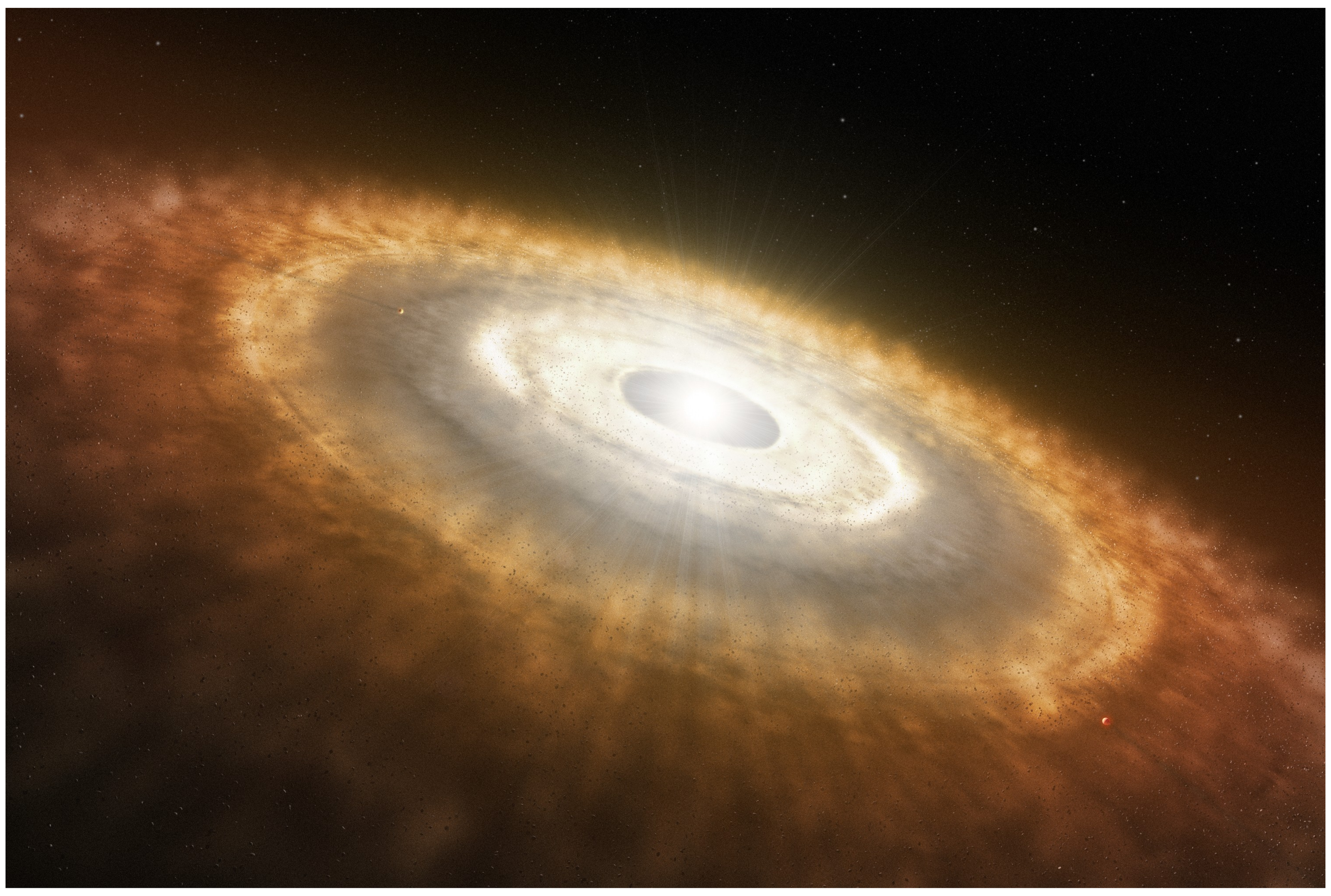
Introduction



(Image credit: ESO)

Planets are built from planetesimals. Planetesimals are built from small dust particles. The growth from 10^{-6}m-sized dust to 10^{3}m-sized planetesimals take place inside gaseous protoplanetary disks around young stars. The interaction between gas and dust plays a critical role in this process. However, traditional methods to study this interaction can be complicated and expensive. We present a novel approach to model dust-gas dynamics by exploiting its physical similarity to existing problems in standard hydrodynamics. This framework allows us to understand complex dust-gas dynamics in a simple way, as well as reduce computation costs in simulating dusty protoplanetary disks.

Dust-gas drag

- One of the most important dust-gas interaction is their mutual friction, e.g.

$$\frac{\partial \vec{v}_{\mathrm{dust}}}{\partial t} = -\frac{(\vec{v}_{\mathrm{dust}} - \vec{v}_{\mathrm{gas}})}{\tau_{\mathrm{stop}}}$$

- τ_{stop} encompasses details of gas drag
- $\tau_{\mathrm{stop}} \to 0$ for small particles $\Rightarrow$ dust carried by gas

Dust+gas: the classic two-fluid approach

A common way to model a system of gas mixed with small solid particles is to treat it as a two phase, continuous medium:

- The gas with density, pressure and velocity $(\rho_{\mathrm{g}}, P, \vec{v}_{\mathrm{g}})$
- The particles as a pressureless fluid with density and velocity $(\rho_{\mathrm{d}}, \vec{v}_{\mathrm{d}})$

$\Rightarrow$ complicated system of equations

(2 for mass conservation, 6 for momentum conservation, 1 energy equation)

A simpler approach

- The equations describing the dust+gas system can be re-formulated to describe a one-fluid mixture (Laibe & Price, 2014, MNRAS).
- We identified a connection between the dusty gas and pure ideal gas

isothermal gas+dust+drag $\longleftrightarrow$ (equivalent for small particles) ideal gas+cooling

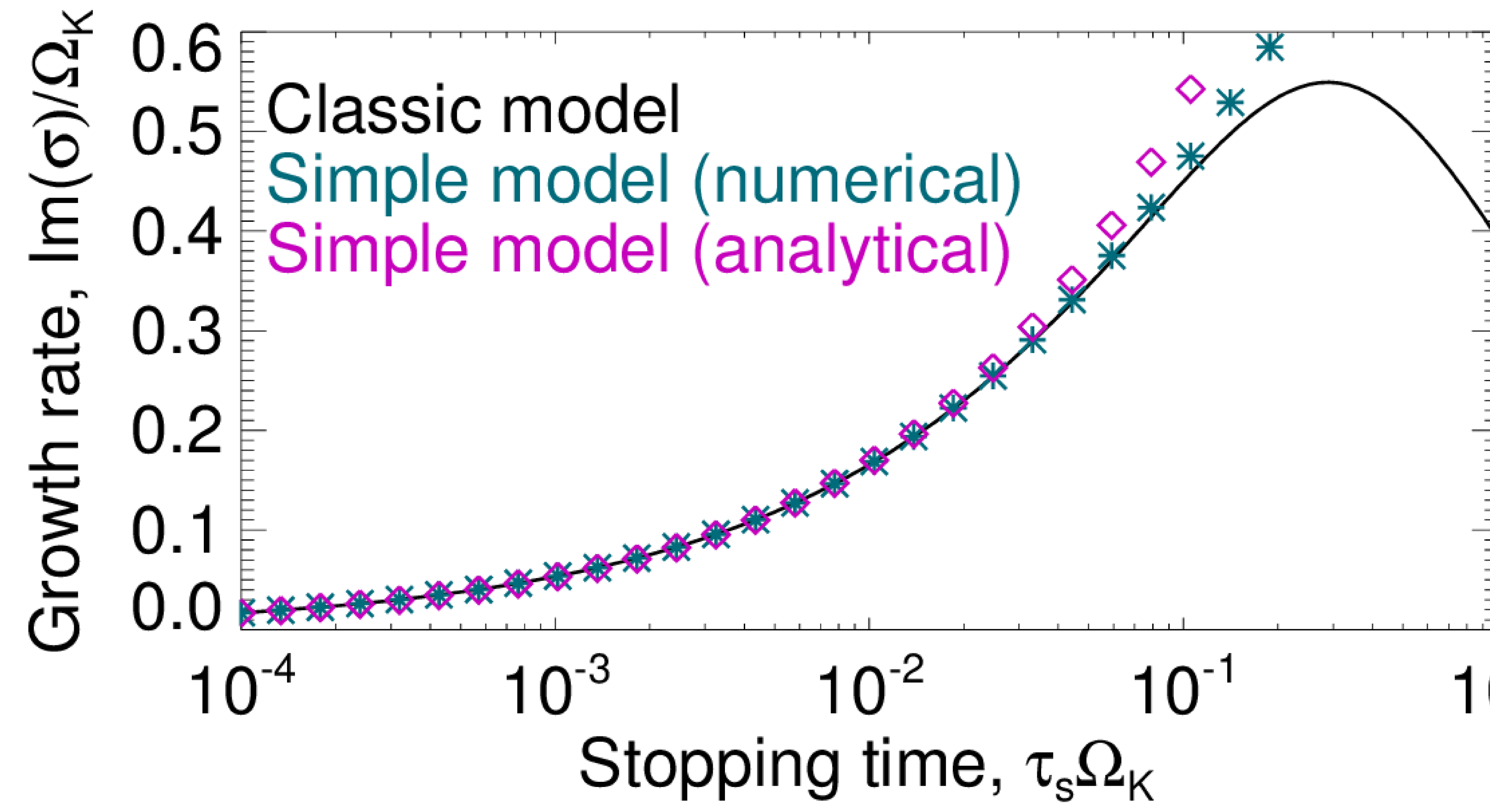
- New model is similar to textbook hydrodynamics

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0,$$
$$\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v} = -\frac{1}{\rho}\nabla P - \nabla\Phi_*,$$
$$\frac{\partial P}{\partial t} + \vec{v}\cdot\nabla P = -P\nabla\cdot\vec{v} + c_s^2\nabla\cdot\left(\frac{\rho_{\mathrm{g}}\rho_{\mathrm{d}}}{\rho^2}\tau_{\mathrm{s}}\nabla P\right).$$

- $\rho =$ total density $\rho_{\mathrm{g}} + \rho_{\mathrm{d}}$
- $\vec{v} =$ center of mass velocity $(\rho_{\mathrm{g}}\vec{v}_{\mathrm{g}} + \rho_{\mathrm{d}}\vec{v}_{\mathrm{d}})/\rho$
- Dust-gas drag is included through a pseudo-energy equation

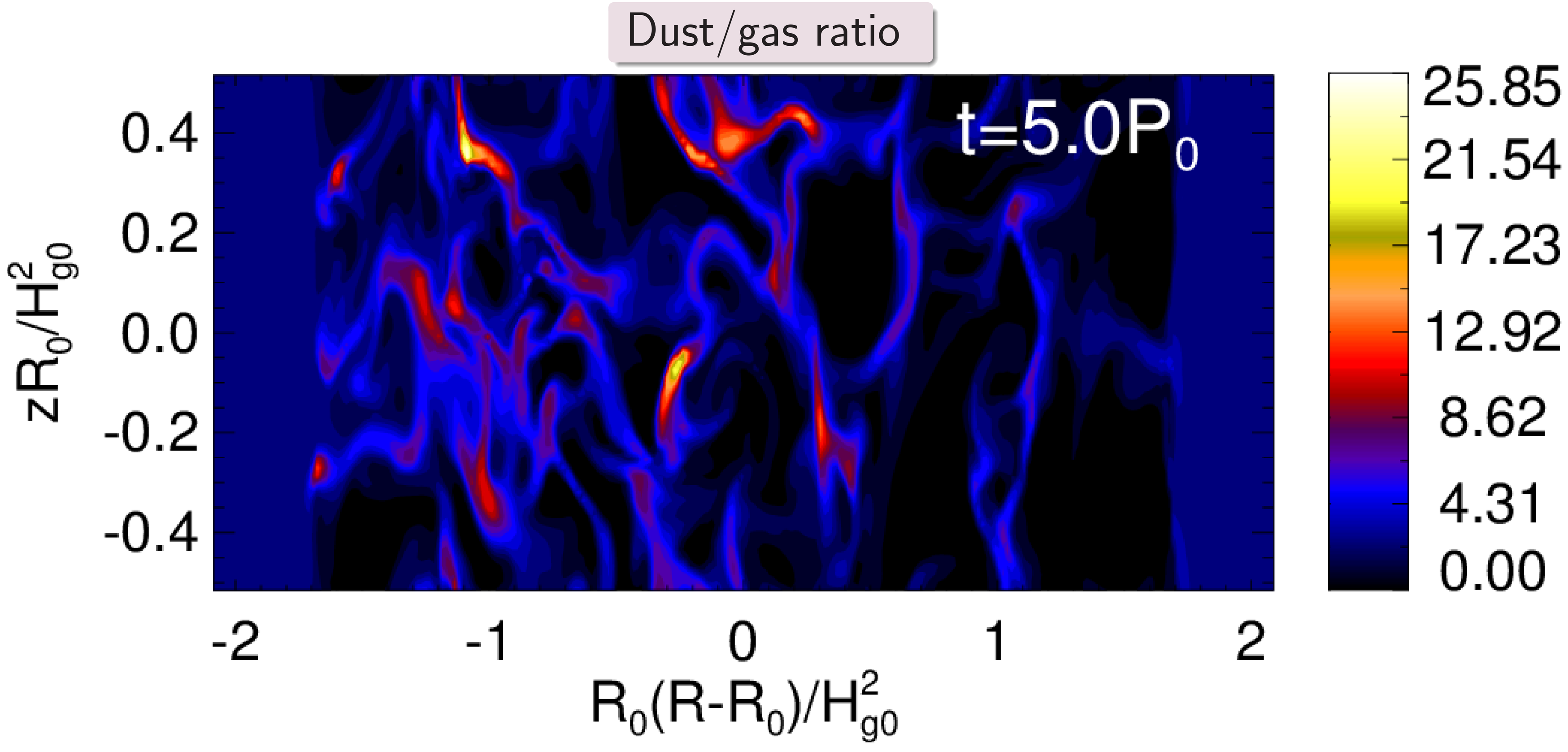
Application I: the streaming instability

- A rotating disk of dust+gas is linearly unstable
- Small disturbances grow like $e^{-i\sigma t}$



- Simplified model correctly captures the instability for small particles

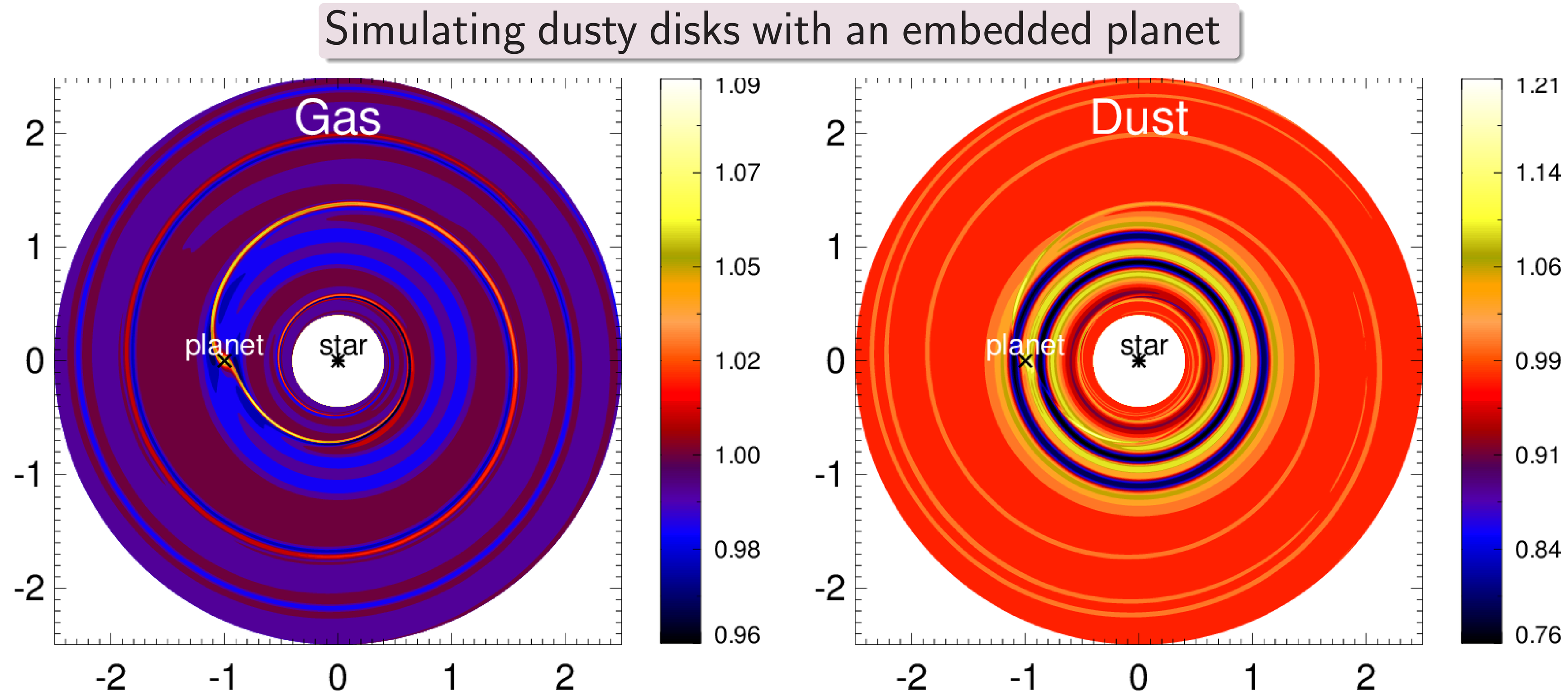
Dust/gas ratio



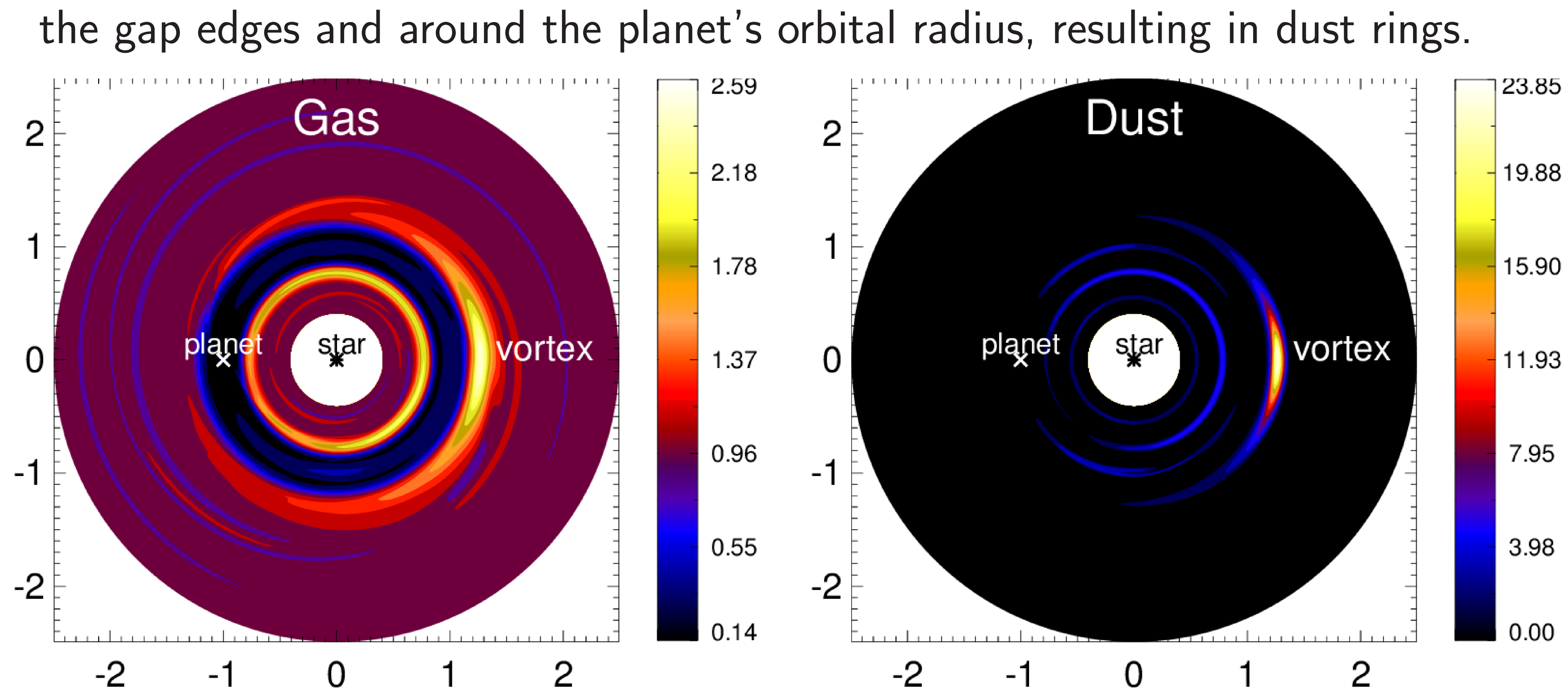
- Streaming instability $\to$ high dust density $\to$ collapse into planetesimals

Application II: dust-trapping

Simulating dusty disks with an embedded planet



- A **3**-Earth mass planet opens up a gas gap in the disk. Dust becomes trapped at the gap edges and around the planet's orbital radius, resulting in dust rings.



- A **30**-Earth mass planet opens up an unstable gas gap, resulting in vortex formation. The vortex traps dust into high concentrations, creating an asymmetric disk. Similar dust asymmetries have also been observed in real disks.

Summary

We find a mapping between dust-gas dynamics and standard hydrodynamics that simplifies the modeling of complex dust-gas interactions. We show this framework reproduces well-known effects in protoplanetary disks, such as the streaming instability and dust-trapping by vortices.

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