Vortices and spirals at gap edges in 3D self-gravitating disk-planet simulations

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September 20 2012
Unstable planetary gaps

- Giant planets in self-gravitating disks.

Low mass disk $\rightarrow$ VORTICES
(Koller et al., 2003; Li et al., 2005; Ou et al., 2007; Li et al., 2009; Yu & Li, 2009; Lin & Papaloizou, 2010, 2011a)

High mass disk $\rightarrow$ SPIRALS
(Meschiari & Laughlin, 2008; Lin & Papaloizou, 2011b, 2012)
Outline

- Review
- Examples in 3D
- Discussion
- Upcoming
Non-axisymmetric instabilities in structured 2D disks

- Potential vorticity (vortensity) extrema is necessary for instability:

\[ \eta \equiv \frac{\kappa^2}{2\Omega\Sigma} \]

- Note: barotropic, non-magnetized
- Nearly-Keplerian disk: \( \kappa \sim \Omega \) so \( \eta \sim \kappa/\Sigma \propto Q \)
Non-axisymmetric instabilities in structured 2D disks

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- Nearly-Keplerian disk: \( \kappa \sim \Omega \) so \( \eta \sim \kappa/\Sigma \propto Q \)
  \[ Q \equiv \frac{c_s \kappa}{\pi G \Sigma} = \frac{c_s}{\pi G} \left( \frac{2\Omega \eta}{\Sigma} \right)^{1/2} \]

- Planet gaps: \( Q \) and \( \eta \) look similar
Planetary gaps in terms of $Q$

- Local min($Q$) $\rightarrow$ vortices
- Local max($Q$) plus $Q(r_{out}) \lesssim 2$ $\rightarrow$ spirals
Checklist for 3D simulations

2D results:
- Vortex formation in low $M_d$, fast merging
- More vortices with increasing $M_d$, resisted/delayed merging
- Spirals with large $M_d$

3D setup:
- Inviscid 3D disk in spherical polars
- Locally isothermal (now with energy equation)
- Self-gravity parametrized by $Q_0 = Q(r_{out})$ or $M_d$
- Giant planet: 1 or 2-Jupiter mass
$Q_0 = \infty$ verses $Q_0 = 8$

- Vortex formation, checked.
$Q_0 = \infty$ verses $Q_0 = 8$

Vortex vertical structure (relative density perturbation)

Initially $Q \sim 10$ in this region $\rightarrow$ but $\Delta \rho/\rho$ is stratified
\[ Q_0 = 4 \text{ and } Q_0 = 3 \]

- More vortices and resisted merging, checked.
- Caution: boundary potential needs \( m > \text{vortices} \)
$Q_0 = 3$

- Unperturbed $Q \sim 4$
- No merging yet
- Most stratified at vortex core
- Unable to identify ‘typical’ vertical flow pattern (cf. anti-cyclonic horizontal flow)
$Q_0 = 1.7$ and $Q_0 = 1.5$

- Spirals in massive disks, checked.
- Sharp $\max(Q)$ circumvents need for low $Q$ locally ($Q_{\text{edge}} \sim 10$), but need exterior disk to feel the edge disturbance
- They provide positive co-orbital torques (outward migration possible)
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Three-dimensionality

- Empirical measure: $\sqrt{v_z^2/(v_R^2 + v_z^2)}$

- Approximately 2D disturbance
- Vertical Mach number $\sim$ few per cent
Discussion

- Confirmed that most 2D results persist in 3D, long term 3D evolution unknown (H. Li’s talk)
- Strongest 3D effect: vertical self-gravity

Speculations:
- Vortex stability (‘flattened’ under its own weight)
- Reduction of vertical boundary effects: avoid complicated physics at upper and lower active layers
  (failed to find linear vortex modes with, e.g. $\delta v_{\parallel} = 0$ or $\delta \rho = 0$ at upper disk boundary)
Back to linear stability

- Linear 3D adiabatic disturbances governed by PDE eigenvalue problem
  \[ U(W) = 0, \]
  where \( W \equiv \delta p/\rho \). (freedom to implement vertical b.c.)

- Equilibrium: \( p \propto \rho^\Gamma \) and \( \gamma \geq \Gamma \geq 1 \).

- Solve for nonhomentropic 3D thin disks with a density bump → Rossby wave instability.

- Thick tori version (harder) done by Frank & Robertson (1988) and Kojima et al. (1989) → clues.

- Difficult in general, but brute force works OK for RWI.
Homentropic verses nonhomentropic

• $\gamma/\Gamma = 1$ (polytrope)
Homentropic verses nonhomentropic

\[ \frac{\gamma}{\Gamma} = 1.8 \]
Linear verses nonlinear

- Strictly isothermal equilibria, \( \gamma = 1.4 \)
More details

References